

Lifting D-instanton zero modes by recombination and background fluxes

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ABSTRACT: We study the conditions under which D-brane instantons in Type II orientifold compactifications generate a non-perturbative superpotential. If the instanton is non-invariant under the orientifold action, it carries four instead of the two Goldstone fermions required for superpotential contributions. Unless these are lifted, the instanton can at best generate higher fermionic F-terms of Beasley-Witten type. We analyse two strategies to lift the additional zero modes. First we discuss the process of instantonic brane recombination in Type IIA orientifolds. We show that in some cases charge invariance of the measure enforces the presence of further zero modes which, unlike the Goldstones, cannot be absorbed. In other cases, the instanton exhibits reparameterisation zero modes after recombination and a superpotential is generated if these are lifted by suitable closed or open string couplings. In the second part of the paper we address lifting the extra Goldstones of $D3$ -brane instantons by supersymmetric three-form background fluxes in Type IIB orientifolds. This requires non-trivial gauge flux on the instanton. Only if the part of the fermionic action linear in the gauge flux survives the orientifold projection can the extra Goldstones be lifted.

KEYWORDS: D-branes, Intersecting branes models, Flux compactifications.

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1. Introduction

Since the recent observation that D-brane instantons in Type II orientifolds can induce an important new class of effective couplings [1–4], a lot of effort has gone into further exploring these and other interesting non-perturbative effects [5–16], with directly related earlier work including [17, 18].

In the case of Type IIA orientifolds with intersecting $D6$ -branes, the relevant non-perturbative objects are Euclidean $D2$ -brane instantons, short $E2$ -instantons, wrapping special Lagrangian three-cycles of the internal Calabi-Yau space [1, 3]. An analysis of the zero mode structure of such instantons can be performed with the help of boundary CFT methods as originally applied to the $D3 - D(-1)$ system in [19, 20]. This has shown that

under suitable circumstances the $E2$ -instanton can generate couplings in the effective four-dimensional superpotential which are forbidden perturbatively as a consequence of global $U(1)$ selection rules. The relevant instanton effect is genuinely stringy in that it cannot be understood in terms of four-dimensional gauge instantons.

The imprints of this phenomenon in various corners of the string landscape are manifold. Of particular phenomenological interest has been the generation of Majorana mass terms for right-handed neutrinos [1, 3, 13–15]. Besides allowing for such terms in the first place, instanton effects admit a natural engineering of the intermediate mass scale required for these Majorana terms in the context of the see-saw mechanism. Other applications include the generation of hierarchically small μ -terms [1, 3] or a modification of the family structure of Yukawa couplings [8]. In [7], the generation of perturbatively forbidden $\mathbf{10} \mathbf{10} \mathbf{5}_H$ couplings in $SU(5)$ GUT models based on intersecting branes is discussed. Globally defined examples of an instanton induced lifting of unwanted chiral exotics are presented in [10, 7]. The benefits of instanton effects for realising metastability and supersymmetry breaking in explicit setups are explored in [4–6].

Using the CFT description for the computation of $E2$ -instanton generated superpotential couplings proposed in [1], the non-perturbative Majorana mass matrix for right-handed neutrinos was determined in detail for a local GUT-like toroidal brane setup in [15]. An extensive search for realisations of this effect within the class [21] of global semi-realistic Gepner model orientifolds has been performed in [13], followed by further phenomenological studies in [14].

The main obstacle for finding appealing global string vacua exhibiting a non-perturbative superpotential of the described type are the severe restrictions on the zero mode structure of the instanton, which will be reviewed in detail in section 2 of this article. At least in the absence of other mechanisms to lift the fermionic zero modes associated with deformations of the cycle, the instanton has to be rigid. Unfortunately, for toroidal orbifolds, a popular playground for Type IIA model building, the only known examples of such cycles are the ones on the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ orbifold analysed in [22–24] and used in the local setup of [15].

A second complication, which is the central topic of this paper, occurs for $E2$ -instantons on non-invariant cycles, called $U(1)$ instantons in the following. It is given by the appearance of four Goldstino modes $\theta^\alpha, \bar{\tau}^{\dot{\alpha}}$, $\alpha, \dot{\alpha} = 1, 2$ instead of the two Goldstinos θ^α required for the generation of a superpotential [11–13]. If the instanton lies on top of an appropriate orientifold plane, the two extra modes $\bar{\tau}^{\dot{\alpha}}$ are projected out and the instanton can induce a superpotential term. Given its significance for the topography of the landscape of string vacua, it is obviously quite important to investigate if this is actually the only configuration of D-brane instantons which induces quantum corrections of the superpotential.

The key point is to decide if there exists a way to lift the two extra Goldstinos $\bar{\tau}^{\dot{\alpha}}$ other than by projecting them out. Generally speaking, this requires contact terms in the instanton moduli action involving the modes $\bar{\tau}^{\dot{\alpha}}$ such that they can be soaked up in the path integral without giving rise to higher derivative or higher fermionic terms in the non-perturbative couplings.

We investigate two different strategies to achieve this. In section 3, we analyse cou-

plings of the $\overline{\tau}^\alpha$ modes to massless states in the $E2 - E2'$ sector, which likewise have to be absorbed. As a consequence of the D-term constraints for the bosonic zero modes the lifting of these modes requires the presence of a non-vanishing Fayet-Iliopoulos term. The latter arises after slightly deforming the background such that the $\Xi - \Xi'$ pair of instantonic branes recombines.

We describe in detail the zero mode structure of the $U(1)$ instantons and how it changes by the process of condensation of the bosonic modes. We find that for chiral $\Xi - \Xi'$ recombination, due to charge conservation the recombined object always contains extra fermionic zero modes which cannot be absorbed by pulling down either closed string or matter fields.

However, for non-chiral $\Xi - \Xi'$ recombination one obtains an $O(1)$ instanton with deformations. In the Type I dual model it corresponds to an $E1$ -instanton which wraps a holomorphic curve moving in a family as discussed by Beasley and Witten in [25]. We show that such instantons can generate, in addition to the results of [25], multi-fermion couplings also for matter field superpotentials and under certain circumstances can also contribute to the superpotential. Independently of the issue of instanton recombination, in the absence of $E2 - E2'$ modes the measure of rigid $U(1)$ instantons is just right to generate possibly open string dependent multi-fermion F-terms which correct the metric on the complex structure moduli space. This is the subject of section 4.

An alternative mechanism to eliminate the $\overline{\tau}^\alpha$ modes, speculated upon already in the literature [12, 13, 6], consists in turning on supersymmetric background fluxes. The hope would be that in their presence the instanton does not feel the full $\mathcal{N} = 2$ supersymmetry algebra preserved locally away from the orientifold, but only the $\mathcal{N} = 1$ subalgebra preserved by the fluxes. This should then result in only two as opposed to four Goldstinos.

The lifting of reparametrisation zero modes of $M5$ -brane or Type IIB $D3$ -brane instantons has been studied in detail [26–31] (see also [32–34]). The analysis consists in determining the bilinear couplings of the fermionic zero modes to the background fluxes responsible for their lifting. In section 5 we recall, building upon the expressions for the fermion bilinears derived in [28, 30], that in Type IIB orientifolds a lifting of the $\overline{\tau}^\alpha$ of $E3$ -instantons is not possible as long as one sticks to supersymmetric three-form flux. As we then show, this generically continues to hold even for $E3$ -instantons with gauge flux which are mirror symmetric to Type IIA $U(1)$ instantons at general angles. A possible exception are compactifications with divisors allowing for anti-invariant two-cycles. We illustrate this point in a local example and finally summarize our findings in section 6.

2. Instanton generated F-terms

We are interested in $\mathcal{N} = 1$ supersymmetric Type II orientifold compactifications to four dimensions. While what we have to say in the sequel applies, *mutatis mutandis*, equally well to Type IIA and Type IIB constructions, we focus here for definiteness on the first case. We will therefore be working in the context of intersecting $D6$ -brane models (see [35–40] for reviews). The relevant spacetime instantons are given by $E2$ -branes wrapping special Lagrangian three-cycles Ξ in the Calabi-Yau so that they are point-like in four-dimensional

$\mathcal{N} = 1$	$\mathcal{N} = 1'$
θ^α	τ^α
$\bar{\theta}^{\dot{\alpha}}$	$\bar{\tau}^{\dot{\alpha}}$

Table 1: Universal fermionic zero modes $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$ ($\tau^\alpha, \bar{\tau}^{\dot{\alpha}}$) of an (anti-)instanton associated with the breaking of the $\mathcal{N} = 1$ SUSY algebra preserved by the orientifold and its orthogonal complement $\mathcal{N} = 1'$.

spacetime. Part of the following two subsections 2.1 and 2.2 reviews some of the findings of [1, 3], while in 2.3 we discuss higher fermionic F-terms.

2.1 $E2$ -instanton zero modes

There are two kinds of instanton zero modes according to their charge under the gauge groups on the $D6$ -branes.

The uncharged zero modes arise from the $E2$ - $E2$ sector. They always comprise the universal four bosonic Goldstone zero modes x^μ due to the breakdown of four-dimensional Poincaré invariance. Generically, for instantons away from the orientifold fixed plane, these come with four fermionic zero modes θ^α and $\bar{\theta}^{\dot{\alpha}}$ [11–13]. This reflects the fact that the instanton breaks half of the eight supercharges preserved by the Calabi-Yau manifold away from the orientifold fixed plane. Due to its localisation in the four external dimensions, an instanton breaks one half of the $\mathcal{N} = 1$ supersymmetry preserved by the orientifold and one half of its orthogonal complement inside the $\mathcal{N} = 2$ supersymmetry algebra preserved by the Calabi-Yau. As displayed in table 1, the θ^α are the Goldstinos associated with the breakdown of the first $\mathcal{N} = 1$ supersymmetry, while the $\bar{\theta}^{\dot{\alpha}}$ are associated with the orthogonal $\mathcal{N} = 1'$ algebra.¹ The internal part of their vertex operator is essentially given by the spectral flow operator of the worldsheet $\mathcal{N} = (2, 2)$ superconformal theory, see eq. (B.9) and (B.1) in appendix B.

Besides there are $b_1(\Xi)$ complex bosonic zero modes $c_I, I = 1, \dots, b_1(\Xi)$, related to the deformations and Wilson lines of the $E2$ -instanton. Away from the orientifold plane, each of these is accompanied by one chiral and one anti-chiral Weyl spinor, χ_I^α and $\bar{\chi}_I^{\dot{\alpha}}$. Furthermore there arise zero modes at non-trivial intersections of the instanton $E2$ with its image $E2'$; they will be discussed in detail in section 3.1.

In addition to these uncharged zero modes, there can arise fermionic zero modes from intersections of the instanton Ξ with $D6$ -branes Π_a . If the instanton is parallel to Π_a , there are also massless bosonic modes in this sector. The detailed quantisation of these charged zero modes, both for chiral and non-chiral intersections, is described in [15]. Let us focus for brevity on chiral intersections. An important point made in [15] is that states in the $E2 - D6$ sector are odd under the GSO projection contrary to the GSO-even states in the $D6 - D6$ brane sector. In particular, a positive intersection $I_{\Xi a} > 0$ of the instanton and a $D6$ -brane wrapping the respective cycles Ξ and Π_a hosts a single *chiral* fermion (i.e. with world-sheet charge $Q_{ws} = -\frac{1}{2}$) in the bifundamental representation $(-1_E, \square_a)$. The strict

¹Note that what was called $\bar{\theta}^{\dot{\alpha}}$ in [15] is now denoted by $\bar{\tau}^{\dot{\alpha}}$ to make its spacetime interpretation clearer.

zero modes	Reps $_{Q_{ws}}$	number
$\lambda_{a,I}$	$(-1_E, \square_a)_{-1/2}$	$I = 1, \dots, [\Xi \cap \Pi_a]^+$
$\bar{\lambda}_{a,I}$	$(1_E, \bar{\square}_a)_{-1/2}$	$I = 1, \dots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \bar{\square}_a)_{-1/2}$	$I = 1, \dots, [\Xi \cap \Pi'_a]^+$
$\bar{\lambda}_{a',I}$	$(1_E, \square_a)_{-1/2}$	$I = 1, \dots, [\Xi \cap \Pi'_a]^-$

Table 2: Fermionic zero modes at chiral $E2 - D6$ intersections for non-invariant $E2$ -instantons.

chirality of the charged fermions is essential for the existence of holomorphic couplings between these modes and open string states in the moduli action and will also play a key role in the present analysis. For a generic instanton cycle Ξ away from the orientifold, this gives rise to the charged zero mode spectrum summarised in table 2. As a result, the instanton carries the charge [1, 3].

$$Q_a(E2) = \mathcal{N}_a \Xi \circ (\Pi_a - \Pi'_a) \tag{2.1}$$

under the gauge group $U(1)_a$.

2.2 Generation of superpotentials

The instanton measure contains all these zero modes. Thus in order to contribute to the holomorphic superpotential, whose measure is $\int d^4x d^2\theta$, the instanton has to meet several constraints.

Most importantly, the presence of the anti-chiral Goldstinos $\bar{\tau}^{\dot{\alpha}}$ for generic instantons not invariant under the orientifold projection² prevents the generation of superpotential terms other than those corresponding to gauge instantons [11–13]. The latter case is special in that the instanton wraps the same three-cycle as one of the $D6$ -branes [9]. In this situation, the $\bar{\tau}^{\dot{\alpha}}$ play the role of Lagrange multipliers for the bosonic ADHM constraints and can consistently be integrated out [20]. For instantons not parallel to any of the $D6$ -branes, these couplings in the moduli action do not exist since there are no massless bosons in the $E2 - D6$ sector. The most straightforward way to eliminate the $\bar{\tau}^{\dot{\alpha}}$ is to project them out under the orientifold action [11–13]. Concretely, if one chooses $\Xi = \Xi'$ the universal zero modes $x^\mu, \theta^\alpha, \bar{\tau}^{\dot{\alpha}}$ are subject to the orientifold action $\Omega\bar{\sigma}$ in the way detailed in appendix A. Depending on the orientifold action one obtains an $SO(N)$ or $USp(N)$ gauge group. For the latter case the zero modes x^μ, θ^α are anti-symmetrised and the modes $\bar{\tau}^{\dot{\alpha}}$ gets symmetrised, while for the $SO(N)$ instanton x^μ, θ^α are symmetrised and $\bar{\tau}^{\dot{\alpha}}$ get anti-symmetrised.

It follows that single $E2$ -instantons with orthogonal gauge group (called $O(1)$ instantons in the sequel) can give rise to F-terms in the effective action since the universal part of their zero mode measure is of the form $\int d^4x d^2\theta$.

In order for this F-term to be of the usual superpotential form, there may be no further uncharged fermionic zero modes present. This situation corresponds to an instanton wrapping a rigid cycle Ξ with $b_1(\Xi) = 0$. Alternatively, the additional fermionic modes

²These will be referred to as $U(1)$ instantons in the sequel.

zero modes	Reps.	number
$\lambda_{a,I}$	\square_a	$I = 1, \dots, [\Xi \cap \Pi_a]^+$
$\bar{\lambda}_{a,I}$	$\bar{\square}_a$	$I = 1, \dots, [\Xi \cap \Pi_a]^-$

Table 3: Fermionic zero modes at chiral $E2 - D6$ intersections for invariant $E2$ -instantons.

have to be absorbed by some interaction in the instanton moduli action such that they can be integrated out without generating higher derivative terms. Known examples of such interactions involving the closed string sector are the quartic coupling to the curvature on the instanton moduli space [41, 42], provided the latter is non-trivial, or the coupling to suitable background fluxes (see section 5). In section 3.4 we will describe another way to lift a pair of reparametrisation modes through couplings to the open string sector.

Finally, also the charged zero modes appear in the measure and have to be soaked up. For an $\Omega\bar{\sigma}$ invariant instanton, i.e. $\Xi' = \Xi$, the charged zero modes and their representations are displayed in table 3 and lead to an instanton $U(1)_a$ charge

$$Q_a(E2) = \mathcal{N}_a \Xi \circ \Pi_a. \tag{2.2}$$

A careful analysis of their g_s scaling in [1, 15] revealed that for superpotential couplings this has to happen via suitable disk (as opposed to higher genus) amplitudes involving precisely two λ modes and in addition suitable matter fields - provided these amplitudes induce a Yukawa-type contact term in the instanton moduli action. As a result, $E2$ -instantons induce superpotential terms of the form [1, 3].

$$W \simeq \prod_{i=1}^M \Phi_{a_i, b_i} e^{-S_{E2}}, \tag{2.3}$$

involving suitable products of open string fields Φ_{a_i, b_i} . For details of the rules of their computation see [1].

2.3 Generation of higher fermionic F-terms

Our discussion has hitherto focussed on $O(1)$ instantons which are either rigid or whose fermionic reparametrisation modes have paired up appropriately such that they give rise to genuine superpotential terms.

Alternatively, there are situations where these additional zero modes induce so-called higher fermionic F-term couplings in the effective action. In the dual Type I/heterotic model this effect was first described in [25].³ There it arises for $E1$ /worldsheet instantons moving in a family. On the type IIA side, this corresponds to non-rigid $O(1)$ instantons such that the chiral reparametrisation moduli $\chi_I^\alpha, I = 1, \dots, b_1(\Xi)$ are anti-symmetrised and therefore projected out under the orientifold action. We will sometimes refer to them as instantons with deformations of the first kind.⁴ The resulting uncharged part of the

³For another example in the context of heterotic M-theory see [43].

⁴This is to be contrasted with the case that the chiral deformation fermions survive the projection. As described in [16] such a situation can generate corrections to the gauge kinetic function.

measure takes the form

$$\int d^4x d^2\theta \prod_I c_I \bar{c}_I \bar{\chi}_I^{\dot{\alpha}}. \tag{2.4}$$

Beasley and Witten found that such instantons can generate higher fermionic couplings for the closed string moduli fields [25]. In superspace notation, these are encapsulated in interactions of the form

$$S = \int d^4x d^2\theta w_{\bar{i}\bar{j}}(\Phi) \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}^{\bar{i}} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}^{\bar{j}} \tag{2.5}$$

for the simplest case that the instanton moves in a one-dimensional moduli space. Note that supersymmetry requires a holomorphic dependence of $w_{\bar{i}\bar{j}}(\Phi)$ on the superfields Φ .

Consider first the case of an $E2$ -instanton with $b_1(\Xi) = 1$ and no further charged zero modes in the $E2 - D6$ sector. Denoting by $\mathcal{T} = T + \theta^\alpha t_\alpha$ the $\mathcal{N} = 1$ chiral superfield associated with the Kähler moduli, we can absorb the instanton modulini by pulling down from the moduli action two copies of the schematic anti-holomorphic coupling $\bar{\chi}^{\dot{\alpha}} \bar{t}_{\dot{\alpha}}$. In general the open-closed amplitude $\langle \bar{\chi}^{\dot{\alpha}} \bar{t}_{\dot{\alpha}} \rangle$ does not violate any obvious selection rule of the $\mathcal{N} = (2, 2)$ worldsheet theory and is therefore expected to induce the above coupling.⁵ Similarly, the two θ -modes can be soaked up by the holomorphic coupling $\theta^\alpha u_\alpha$ involving the fermionic partners of the complex structure moduli encoded in the superfield $\mathcal{U} = U + \theta^\alpha u_\alpha$. This results in a four-fermion interaction of the schematic form $e^{-S_{E2}} u^\alpha u_\alpha \bar{t}^{\dot{\alpha}} \bar{t}_{\dot{\alpha}}$. Note that the coupling of the complex and Kähler structure modulini only to the universal and reparametrisation zero modes, respectively, is a consequence of $U(1)$ worldsheet charges of the associated vertex operators.

The derivative superpartner of the above four-fermi term arises upon integrating out two copies of the term

$$\theta \sigma^\mu \bar{\chi} \partial_\mu \bar{\mathcal{T}}, \tag{2.6}$$

which follows from evaluating the amplitude $\langle \theta^\alpha \bar{\chi}^{\dot{\alpha}} \bar{\mathcal{T}} \rangle$ as demonstrated in appendix B. All this can be summarized in superspace notation by writing

$$S = \int d^4x d^2\theta e^{-\mathcal{U}(\Xi)} f_{\bar{i}\bar{j}}(e^{\mathcal{T}_i}, e^{\Delta_i}) \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\mathcal{T}}^{\bar{i}} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{T}}^{\bar{j}}, \tag{2.7}$$

where $\mathcal{U}(\Xi)$ is associated with the specific combination of complex structure moduli appearing in the classical instanton action and the holomorphic function $f_{\bar{i}\bar{j}}$ depends in general on the Kähler and open string moduli of the $D6$ -branes Δ_i .

In the presence of a suitable number of charged λ zero-modes there exist, in addition to these closed string couplings, terms which generate higher fermi-couplings also for the matter fields. Consider again for simplicity the case $b_1(\Xi) = 1$. If the Chan-Paton factors

⁵In particular, the *total* $U(1)$ worldsheet charge is conserved. Still there might be situations, such as factorizable 3-cycles on $(T^2)^3$, where some of the *individual* $U(1)$ charges are violated by this coupling. For a generic background, though, the couplings need not be vanishing, as we demonstrate for the example of a non-factorizable T^6 in appendix B.

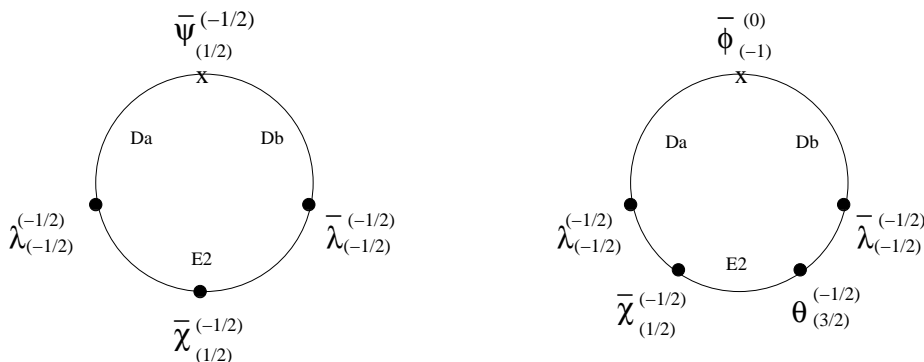


Figure 1: Absorption of θ and $\bar{\chi}$ -modes leading to F-terms. The superscripts denote the ghost picture.

and worldsheet selection rules only allow the λ modes to couple holomorphically to the chiral open string superfields, as for the generation of a superpotential, the instanton induces an interaction as in (2.7), but with $e^{-\mathcal{U}(\Xi)}$ replaced by $e^{-\mathcal{U}(\Xi)} \prod_{a_i, b_i} \Phi_{a_i, b_i}$.

For suitable configurations, the action can also pick up derivative terms directly involving the open string fields. For this to happen the instanton moduli action has to contain couplings of the form⁶

$$\bar{\chi}_{1/2}^{\dot{\alpha}} \lambda_{-1/2}^a (\bar{\psi}_{1/2})_{\dot{\alpha}} \bar{\lambda}_{-1/2}^b, \quad (2.8)$$

where the fermionic matter field $\bar{\psi}_{1/2}^{\dot{\alpha}}$ lives at the intersection $D6_a - D6_b$ and lies in the anti-chiral superfield $\bar{\Phi} = \bar{\phi} + \bar{\tau}\bar{\psi}$, see figure 1.

Integrating out two copies of this interaction term brings down the fermion bilinear $\bar{\psi}_{1/2} \bar{\psi}_{1/2}$. In addition, the two θ^α modes again pull down a bilinear of *chiral* fermions u^α or, in the presence of more λ modes, ψ_{ab}^α , as in the case of superpotential contributions. This induces again a four-fermi coupling. Alternatively, we can absorb one pair of $\theta^\alpha \bar{\chi}^{\dot{\alpha}}$ in a coupling of the form

$$\theta_{3/2}^\alpha \bar{\chi}_{1/2}^{\dot{\alpha}} \lambda_{-1/2}^a \bar{\phi}_{-1} \bar{\lambda}_{-1/2}^b. \quad (2.9)$$

After bringing the $\bar{\phi}_{-1}$ into the zero ghost picture this clearly generates a derivative coupling of the form $\theta \sigma^\mu \bar{\chi} \lambda^a \partial_\mu \bar{\phi} \bar{\lambda}^b$. Integrating out two copies of this term yields the derivative superpartner to the above four-fermi term.

3. Instanton recombination

As just reviewed, for the case of $E2$ -instantons in Type IIA orientifolds we know that single instantons wrapping rigid special Lagrangian three-cycles invariant under the orientifold

⁶The subscripts denote the worldsheet $U(1)$ -charges, which are obviously conserved. The actual presence of this contact term can easily be checked, in the context of toroidal orbifolds, by a computation analogous to those performed in [15].

projection and carrying $O(1)$ gauge group have the right zero mode structure $\int d^4x d^2\theta$ to contribute to the superpotential. Under mirror symmetry to the Type I string these objects are mapped to $E1$ -instantons wrapping isolated curves on the mirror Calabi-Yau. The contribution of such objects to the superpotential has been discussed in a couple of papers [44, 25].

For $D6$ -branes it is known that under certain circumstances a pair of $D6$ - $D6'$ branes can recombine [45] into a new sLag $D6$ -brane which obviously wraps an $\Omega\bar{\sigma}$ invariant three-cycle.⁷ If a similar story also applies to pairs of $E2$ - $E2'$ instantonic branes, the recombined objects would be candidates for new $O(1)$ -instantons contributing to the superpotential. For example if one starts with an $E2$ -instanton wrapping a factorizable cycle on a toroidal orbifold, the cycle wrapped by the recombined instanton would no longer be factorizable; still one could hope to determine the instanton contribution by appropriate deformation of the original instanton moduli action. In the mirror dual situation, the resulting objects are $E5$ -instantons equipped with a vector bundle W defined via the non-trivial extension

$$0 \rightarrow L \rightarrow W \rightarrow L^* \rightarrow 0 \tag{3.1}$$

of the two line bundles L and L^* .

In this section we investigate whether the naive expectation that such recombined $O(1)$ -instantons exist is actually correct.

3.1 Zero mode structure on $U(1)$ instantons

Consider a $U(1)$ -instanton wrapping a general rigid cycle $\Xi \neq \Xi'$. From the $E2 - E2$ and $E2' - E2'$ sectors we now have the zero mode measure

$$\int d^4x d^2\theta d^2\bar{\tau}. \tag{3.2}$$

As described in the previous section, if such an instanton also intersects the $D6$ -branes present in the model, this yields the fermionic zero-modes listed in table 2. From there, the overall $U(1)_E$ charge of these matter zero modes can be read off,

$$\begin{aligned} \sum_i Q_E(\lambda^i) &= \sum_a N_a \left(-(\Xi \cap \Pi_a)^+ + (\Xi \cap \Pi_a)^- - (\Xi \cap \Pi_{a'})^+ + (\Xi \cap \Pi_{a'})^- \right) \\ &= - \sum_a N_a \Xi \circ (\Pi_a + \Pi_{a'}) = -4 \Xi \circ \Pi_{O6}. \end{aligned} \tag{3.3}$$

In the last line we have used the tadpole cancellation condition.⁸ This shows that in a globally consistent model the total $U(1)_E$ charge of all matter zero modes is proportional to the chiral intersection between the instanton and the orientifold plane. For an $\Omega\bar{\sigma}$ invariant instanton this last quantity vanishes, whereas for a generic $U(1)$ instanton it does not.

⁷For brane recombination in the context of $D6$ -brane model building see e.g. [46–48].

⁸Notice that Π_{O6} denotes the total homological charge of all orientifold fixed planes present in the background. In what follows we will always refer to the effective orientifold projection which arises after taking into account the contribution from all different sectors, which may be of different types individually.

zero mode	$(Q_E)_{Q_{ws}}$	Multiplicity
m, \bar{m}	$(2)_1, (-2)_{-1}$	$\frac{1}{2} (\Xi' \circ \Xi + \Pi_{O6} \circ \Xi)$
$\bar{\mu}^{\dot{\alpha}}$	$(-2)_{1/2}$	$\frac{1}{2} (\Xi' \circ \Xi + \Pi_{O6} \circ \Xi)$
μ^α	$(2)_{-1/2}$	$\frac{1}{2} (\Xi' \circ \Xi - \Pi_{O6} \circ \Xi)$

Table 4: Charged zero modes at an $E2 - E2'$ intersection.

If $\Xi \circ \Pi_{O6} \neq 0$, there must be additional charged zero modes in order for the zero mode measure to be $U(1)_E$ invariant. Indeed there are also zero modes from the $E2 - E2'$ intersection. This is the open string sector which is invariant under $\Omega\bar{\sigma}$ and gets symmetrized or anti-symmetrized (see appendix A). Taking into account that the sign of the orientifold projection changes from $Dp-Dp$ to $D(p-4)-D(p-4)$ sectors, for a single $U(1)$ instanton we get the zero modes shown in table 4.

For concreteness we consider from now on the two simplest non-trivial cases.

Case I

The first case has intersection numbers

$$\Xi' \circ \Xi = \Pi_{O6} \circ \Xi = 1. \tag{3.4}$$

It corresponds to a projection as would arise e.g. on T^6/\mathbb{Z}_2 in the presence of a single O^- -plane. We get two additional bosonic zero modes m and \bar{m} and two additional fermionic ones $\bar{\mu}^{\dot{\alpha}}$. Comparing with (3.3), we find that indeed the total $U(1)_E$ charge of zero modes vanishes. The charge of the two $\bar{\mu}^{\dot{\alpha}}$ zero modes precisely cancels against the sum over all matter field zero modes.

This analysis reveals that in a globally consistent model it is not possible to wrap an $E2$ -instanton on a cycle $\Xi \neq \Xi'$ without picking up additional charged zero modes λ_i . Their $U(1)_E$ charge is guaranteed to cancel the $U(1)_E$ charge of the $E - E'$ modes such that the resulting zero mode measure,⁹

$$\int d\mathcal{M}_I = \int d^4x d^2\theta d^2\bar{\tau} dm d\bar{m} \underbrace{d^2\bar{\mu}^{\dot{\alpha}}}_{Q_E=4} \underbrace{\prod_b d\bar{\lambda}_b}_{Q_E=-4} \tag{3.5}$$

is $U(1)_E$ invariant.

Case II

The second case has intersection numbers

$$\Xi' \circ \Xi = 1, \quad \Pi_{O6} \circ \Xi = -1. \tag{3.6}$$

Here we get no extra bosonic zero modes and only the two fermionic ones μ^α . Unlike the previous case, this is due to a projection as would arise e.g. in the presence of

⁹Note the inverse scaling behaviour of the Grassmann numbers.

a single O^+ -plane. In such a situation it is not possible to cancel the tadpoles in a supersymmetric way. Nonetheless, we can perform a similar zero mode analysis. Again the condition (3.3) tells us that there are extra fermionic matter zero modes whose $U(1)_E$ charge is equal to $Q_E = -4$. The resulting zero mode measure reads

$$\int d\mathcal{M}_{\text{II}} = \int d^4x d^2\theta d^2\bar{\tau} \underbrace{d^2\mu^\alpha}_{Q_E=-4} \underbrace{\prod_a d\lambda_a}_{Q_E=4}. \tag{3.7}$$

3.2 Recombination of chiral $E2 - E2'$ instantons

The question we would like to address now is whether one can absorb the zero modes for the $U(1)$ instantons in such a way that contributions to the superpotential W are generated. The expectation that this might be the case arises from the analogous situation for intersecting $D6$ -branes, where a slight deformation of the complex structure moduli induces a non-vanishing Fayet-Iliopolous term on the $D6$ -worldvolume leading to condensation of the tachyonic charged matter fields [45]. This brane recombination process preserves the topological charge of the intersecting $D6 - D6'$ branes and therefore yields a supersymmetric brane wrapping a three-cycle which is invariant under $\Omega\bar{\sigma}$.

Consider first the *case I* from the last section. Here we have the bosonic zero modes m and \bar{m} , which appear in a D-term potential of the form

$$S_{E2} = (2m\bar{m} - \xi)^2. \tag{3.8}$$

The complex structure dependent Fayet-Iliopoulos parameter ξ is proportional to the angle modulo 2 between the cycle Ξ and its image Ξ' and vanishes for supersymmetric configurations. Starting from a supersymmetric situation with $\xi = 0$ one can always deform the complex structure to obtain $\xi < 0$ or $\xi > 0$ at least for small ξ . Since the geometry of the internal cycle is independent of whether it is wrapped by a $D6$ -brane or an $E2$ -instanton, we argue that the FI term is forced upon us even in the absence of four-dimensional dynamical gauge fields associated with the abelian gauge group on the instanton.

The D-term constraint resulting from (3.8) is

$$m\bar{m} = \frac{1}{2}\xi \tag{3.9}$$

and has to be implemented by a delta function in the instanton measure. It is useful to parametrise the complex boson m as

$$m = |m| e^{i\alpha}. \tag{3.10}$$

Note that the D-flatness condition as such does not constrain the phase α . The latter can be absorbed by fixing the gauge with respect to the $U(1)_E$ symmetry under which the instanton measure (3.5) is invariant.

It follows that the bosonic part of the instanton measure takes the form

$$\int d^4x d|m| |m| \delta(|m|^2 - \xi/2). \tag{3.11}$$

As ξ becomes positive, the bosonic m modes get tachyonic, signalling an instability towards condensation of the tachyon such that the D-term constraint is satisfied. In the *upstairs* geometry this corresponds to recombination of the cycle $\Xi \cup \Xi'$ (recall that upstairs Ξ and Ξ' are not identified) to the *unique* sLag $\tilde{\Xi}$ with homology class equal to $[\Xi] + [\Xi']$ and $\xi_{\text{new}} = 0$ [49]. Note that $\tilde{\Xi}$ is rigid if Ξ (and Ξ') is rigid [49], i.e. the instanton wrapping it exhibits no uncharged zero modes apart from the universal ones.

Now we have to determine what happens to the fermionic zero modes once the bosonic ones condense. As our analysis of the relevant amplitudes in appendix B shows, the instanton moduli action contains the term

$$S_{E2} = m \bar{\tau}_{\dot{\alpha}} \bar{\mu}^{\dot{\alpha}}, \tag{3.12}$$

which means that after m gets a VEV the $\bar{\tau}$ and $\bar{\mu}$ modes pair up. After bringing down two copies of this terms and integrating out the fermionic zero modes, one is left with the measure

$$\int d\mathcal{M}_I = \int d^4x d^2\theta \prod_a d\lambda_a \prod_b d\bar{\lambda}_b \int d|m| |m|^3 \delta(|m|^2 - \xi/2). \tag{3.13}$$

This is encouraging as with the $\bar{\tau}$ -modes dropping out everything seems to point towards a superpotential contribution. It only remains to absorb the matter zero modes $\lambda_a, \bar{\lambda}_b$ which were forced upon us by $U(1)_E$ invariance of the zero mode measure. Recall that the sum of all charges of these fields is $Q_E = \sum_a Q_E(\lambda_a) + \sum_b Q_E(\bar{\lambda}_b) = 4$. It is clear that pairs of such zero modes with opposite $U(1)_E$ charge can generate the usual matter field couplings of the type

$$\lambda_a \phi_{ab} \bar{\lambda}_b, \tag{3.14}$$

but there will always be the surplus of four zero modes of type $\bar{\lambda}_b$.

As shown in figure 2,¹⁰ due to the $U(1)_E$ charge the only way to absorb these extra $\bar{\lambda}$ zero modes is via couplings of the type

$$\bar{m}^{-1} \bar{\lambda}_b^{-1/2} \prod \phi_{b_i c_i}^1 \bar{\lambda}_c^{-1/2} \tag{3.15}$$

always involving the field \bar{m} . In (3.15) the upper index indicates the world-sheet charge Q_{ws} . Since all the fields except \bar{m} are chiral (in the sense of the $\mathcal{N} = 2$ world-sheet supersymmetry) and \bar{m} itself is anti-chiral, the chiral ring structure tells us that all couplings of type (3.15) are *vanishing*: When we apply the picture changing operator to \bar{m}^{-1} we do not pick up the right pole structure for a non-zero amplitude [50]. On the other hand, with no additional matter field ϕ in (3.15), the amplitude is vanishing right away due to violation of the $U(1)$ world-sheet charge.

Therefore, we conclude that in contrast to naive expectations, the recombined $E2' - E2$ instanton cannot contribute to the superpotential. There always remain four charged fermionic zero modes which cannot be absorbed in a chiral manner.

¹⁰Figure 2 displays the case with no additional matter field, namely $\langle \bar{m} \bar{\lambda} \bar{\lambda} \rangle$.

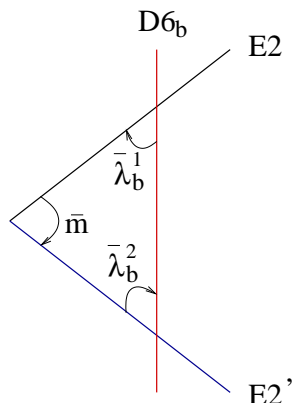


Figure 2: Absorption of $\bar{\lambda}_b$ zero modes.

For *case II* there are no bosonic zero modes from the $E2' - E2$ intersection and therefore no brane recombination. One only has the fermionic zero modes μ^α of $U(1)_E$ charge $Q_E = 2$. In this case we can write down the four-fermion coupling

$$\theta_\alpha^{3/2} (\mu^\alpha)^{-1/2} \lambda_a'^{-1/2} \lambda_a^{-1/2}, \tag{3.16}$$

where again upper indices denote the $U(1)_{ws}$ charges. Therefore, two such couplings can absorb the eight appearing zero modes $\theta, \mu, \lambda_a^i, (\lambda_a')^i$ so that one is only left with the measure

$$\int d\mathcal{M}_{II} = \int d^4x d^2\bar{\tau} \prod_a d\lambda_a \prod_b d\bar{\lambda}_b, \tag{3.17}$$

where the total $U(1)_E$ charge of all the matter zero modes λ_a and $\bar{\lambda}_b$ vanishes. There is no way to absorb the remaining $\bar{\tau}$ modes involving open string operators: Clearly no superpotential terms are generated, as couplings like

$$(\bar{\tau}_{\dot{\alpha}})^{-3/2} \bar{\lambda}_a^{-1/2} (\psi_{ab}^\alpha)^{-1/2} \lambda_b^{-1/2} \tag{3.18}$$

are not allowed by Lorentz invariance and non-holomorphic interactions of the form

$$(\bar{\tau}_{\dot{\alpha}})^{-3/2} \bar{\lambda}_a^{-1/2} (\bar{\psi}_{ab}^\alpha)^{1/2} \lambda_b^{-1/2} \tag{3.19}$$

vanish as a consequence of $U(1)$ worldsheet charge violation. By contrast, it is possible to absorb the $\bar{\tau}$ - modes through couplings to anti-chiral fermions in the closed string sector of the form $\langle \bar{\tau}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} \rangle$, which will be discussed in section 4. Clearly, the induced interactions are non-holomorphic and thus non-supersymmetric. This is however no wonder since, as we recall, the very presence of the effective O^+ -plane leading to this kind of orientifold projection does not admit supersymmetric tadpole cancellation.

We conclude that in contrast to expectations based on spacetime-filling brane recombination processes, instanton recombination does not lead to new $O(1)$ instantons which can contribute to the superpotential. The reason is that due to $U(1)_E$ charge conservation and

zero mode	$(Q_E)_{Q_{ws}}$
m, \bar{m}	$(2)_1, (-2)_{-1}$
$\bar{\rho}^{\dot{\alpha}}$	$(-2)_{1/2}$
n, \bar{n}	$(-2)_1, (2)_{-1}$
$\bar{\nu}^{\dot{\alpha}}$	$(2)_{1/2}$

Table 5: Charged zero modes at non-chiral $E2 - E2'$ intersections on top of an $O6^-$ -plane.

the tadpole cancellation conditions there arises a net number of charged fermionic matter zero modes which cannot be absorbed by chiral couplings.

For the Type IIB dual orientifold models this observation implies that magnetised $E5 - E5'$ recombination, i.e. instantons carrying extensions (3.1) of line bundles, do not generate superpotential contributions either. The only known contributions in this case come from $E1$ -instantons wrapping holomorphic curves on the mirror Calabi-Yau manifold.

3.3 Recombination of non-chiral $E2 - E2'$ instantons

The deeper reason why chiral $E2 - E2'$ intersecting instantons as in *case I* do not lead, after brane recombination, to $O(1)$ instantons seems to be that this $E2 - E2'$ system carries charge along the “directions” of the orientifold $O6^-$ plane. In the Type IIB dual situation this means that the magnetised $E5 - E5'$ system carries $E5$ -brane charge.

Consequently, it may be more promising to start with a magnetised $E3 - \bar{E}3'$ system which after brane-recombination only contains $E1$ -charge. Such a system necessarily has $E3 \circ \bar{E}3' = 0$ and can only support vector-like zero modes on the intersection. This immediately implies that there are no $U(1)_E$ charged matter zero modes necessary to ensure $U(1)_E$ invariance of the zero mode measure.

The simplest non-trivial case involves one vector-like pair of zero modes, i.e.

$$[\Xi' \cap \Xi]^+ = [\Xi' \cap \Xi]^- = 1, \quad [\Pi_{O6} \cap \Xi]^+ = [\Pi_{O6} \cap \Xi]^- = 1. \quad (3.20)$$

Therefore, for an $O6^-$ -plane we have the zero modes shown in figure 5.

There is still the fermionic coupling

$$S_{E2} = \bar{\tau}_{\dot{\alpha}} (m \bar{\rho}^{\dot{\alpha}} - n \bar{\nu}^{\dot{\alpha}}) \quad (3.21)$$

so that the $\bar{\tau}^{\dot{\alpha}}$ modes absorb one linear combination of the fermionic zero modes. In addition the single real bosonic D-term constraint¹¹

$$m_1^2 \bar{m}_{-1}^{-2} - n_1^{-2} \bar{n}_{-1}^2 = 0 \quad (3.22)$$

fixes

$$m \bar{m} = n \bar{n}, \quad (3.23)$$

¹¹One might expect that similar to the ADHM construction of gauge instantons one has three D-term constraints. But from the $U(1)_E$ and $U(1)_{ws}$ charges in table 5 it is clear that one can build only the neutral combination in eq. (3.22).

where the lower index denotes the $U(1)_{\text{ws}}$ charge while the upper one refers to $U(1)_E$. For initially rigid instantons, i.e. in the absence of $E2$ -reparametrisation moduli, there exist no F-term constraints which would prevent a non-vanishing VEV $m \bar{m} = n \bar{n} \neq 0$ corresponding to brane recombination.

As in the analogous process for chiral intersections, recombination breaks the $U(1)_E$. The associated gauge degree of freedom can be used to set

$$m = n \tag{3.24}$$

as opposed to merely (3.23). Integrating out the $\bar{\tau}$ modes together with the linear combination $\bar{\mu} = \bar{\rho} - \bar{\nu}$ of fermionic zero modes as appearing in (3.21) brings down a factor of m^2 .

After recombination, one is left with the measure

$$\int d\mathcal{M}_{\text{III}} = \int d^4x d^2\theta d^2\bar{\mu}_{1/2}^{\bar{\alpha}} dm_1 d\bar{m}_{-1} m_1^2, \tag{3.25}$$

where again the lower index denotes the $U(1)_{\text{ws}}$ charge in the canonical ghost picture and $\bar{\mu}_{1/2}^{\bar{\alpha}} = \bar{\rho} + \bar{\nu}$ stands for the remaining linear combination of fermionic zero modes. In addition, there can of course be charged zero modes $\lambda_a, \bar{\lambda}_b$.

Ignoring the additional factor of m_1^2 for the moment, this zero mode structure is precisely that of an $O(1)$ instanton with one deformation $b_1(\Xi) = 1$ of the first type (see discussion around (2.4)). From our discussion in section 2.3 we expect this configuration to generate higher fermionic F-terms of Beasley-Witten type.

Extrapolating from the CFT of the $E2 - E2'$ sector before recombination, the relevant couplings after recombination are inherited from

$$(\bar{m}_{-1} \bar{\nu}_{1/2}^{\bar{\alpha}} + \bar{n}_{-1} \bar{\rho}_{1/2}^{\bar{\alpha}}) \lambda_{-1/2}^a (\bar{\psi}_{1/2})_{\bar{\alpha}} \bar{\lambda}_{-1/2}^b \longrightarrow \bar{m}_{-1} \bar{\mu}_{1/2}^{\bar{\alpha}} \lambda_{-1/2}^a (\bar{\psi}_{1/2})_{\bar{\alpha}} \bar{\lambda}_{-1/2}^b, \tag{3.26}$$

where the fermionic matter field $\bar{\psi}_{1/2}^{\bar{\alpha}}$ lives at the intersection $D6_a - D6_b$ and lies in the anti-chiral superfield $\bar{\Phi} = \bar{\phi} + \bar{\tau}\bar{\psi}$. Note that the above coupling does not violate any of the general $\mathcal{N} = 2$ SCFT selection rules so that even without a direct computation we expect it to be present for sufficiently generic backgrounds. Integrating out two copies of this interaction term brings down the fermion bilinear $\bar{\psi}_{1/2} \bar{\psi}_{1/2}$ characteristic for the higher fermionic terms described in [25] as well as a factor of \bar{m}_{-1}^2 . The bosonic measure can then be brought into standard form by a simple change of variables with $\tilde{m} = m^3$ and we are left with

$$\int d\mathcal{M}_{\text{III}} = \int d^4x d^2\theta d\tilde{m}_1 d\bar{\tilde{m}}_{-1} \bar{\psi}_{1/2} \bar{\psi}_{1/2}. \tag{3.27}$$

Together with the chiral fermion bilinear pulled by the two θ^α modes this results in the four-fermi terms as discussed in section 2.3.

Its bosonic derivative superpartner involves absorbing one pair of $\theta\bar{\mu}$ in a coupling of the form (after recombination)

$$\bar{m}_{-1} (\bar{\tau}_{\bar{\alpha}})_{3/2} \bar{\mu}_{1/2}^{\bar{\alpha}} \lambda_{-1/2}^a \bar{\phi}_{-1} \bar{\lambda}_{-1/2}^b. \tag{3.28}$$

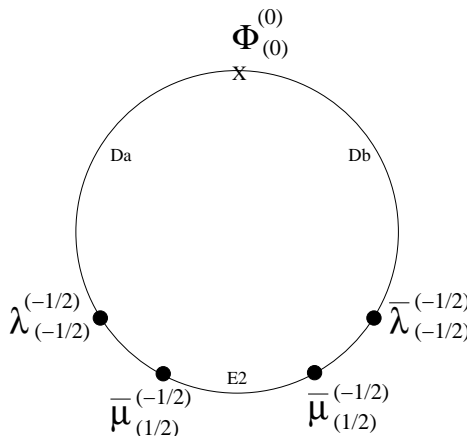


Figure 3: Absorption of $\bar{\mu}_{1/2}^\alpha$ for superpotential contributions. The upper indices are the ghost sectors and the lower ones the Q_{ws} charges.

With $\bar{\phi}_{-1}$ and \bar{m}_{-1} in the zero ghost picture¹² this generates a derivative for the boson $\bar{\phi}_{-1}$. Bringing down two copies of this term indeed yields the derivative superpartner to the above four-fermi term, again in agreement with [25].

3.4 Contribution to superpotential

It has been observed for world-sheet instantons in the heterotic string that instantons moving in a family not only generate higher fermionic F-terms, but can also contribute to the superpotential [41]. Recall that such instantons are dual to $E2$ -instantons with deformations of the first kind and with a zero mode structure as in (2.4) for each deformation. As we just saw, recombination of a non-chiral $E2$ - $E2'$ pair yields precisely such objects. For superpotential contributions to exist it must be possible to absorb the fermionic zero modes without generating higher fermionic or derivative terms as in (3.26) or (3.28). A way to do this for matter field superpotential contributions is shown in figure 3. There $\bar{\mu}$ denotes the fermionic reparameterisation mode independently of whether the instanton is the result of recombination or not. In the first case, we should actually replace $\bar{\mu}$ by $\bar{m}_{-1}\bar{\mu}$ as before.

If this five point function has a contact term and if the remaining integral over the bosonic instanton moduli space does not vanish, then a contribution to the superpotential can be generated. We stress again that from a general $\mathcal{N} = 2$ SCFT point of view, no obvious selection rules forbid such an interaction term. Having said this, one can easily convince oneself that for factorizable three-cycles on toroidal orbifolds the amplitude vanishes due to violation of the $U(1)$ worldsheet charge which has to be conserved for each of the three tori separately. This, however, need not be so for more general setups.

By contrast, it is clear that these disc amplitudes vanish for $E2$ -deformations of the second kind as defined in [16]. Recall from section 2.3 that these give rise to chiral instead

¹²Note that for \bar{m}_{-1} the PCO can only act non-trivially in the internal part since its vertex does not carry any momentum.

of anti-chiral deformation moduli.

To summarize, non-chiral $E2 - E2'$ recombination results in an object with at least two bosonic and two fermionic zero modes from a surviving deformation of the first kind of the recombined instanton. These objects can generate higher fermion couplings and under certain circumstances can also contribute to the superpotential.

4. F-term correction to complex structure moduli space

Having analysed the consequences of zero modes in the $E2 - E2'$ sector in addition to the four Goldstinos for a $U(1)$ instanton, in this section we are interested in the induced couplings if the uncharged measure merely takes the form

$$\int d^4x d^2\theta d^2\bar{\tau} \tag{4.1}$$

in the first place. Consider therefore a rigid $U(1)$ instanton with the geometric intersection numbers

$$\Xi' \cap \Xi = 0 = \Pi_{O6} \cap \Xi. \tag{4.2}$$

This is easily realised e.g. for cycles parallel to, but not on top of the orientifold plane in some subspace. The uncharged zero mode measure (4.1) is to be supplemented by additional charged zero modes λ if present. Since there are no zero modes in the $E2 - E2'$ sector which would be sensitive to the orientifold action, we might expect this type of instantons to be describable in terms of half-BPS instantons of the underlying $\mathcal{N} = 2$ supersymmetry preserved by the internal Calabi-Yau before orientifolding. The correction to the complex structure moduli space metric by $E2$ -instantons in type IIA Calabi-Yau compactification has been discussed recently in [51].¹³ Following this logic, we would anticipate the generation of $E2$ -corrections to the complex structure Kähler potential by the $U(1)$ instanton described by (4.1).

However, while the chiral Goldstino modes θ are indeed associated with the breakdown of the $\mathcal{N} = 1$ subalgebra of this $\mathcal{N} = 2$ symmetry which is preserved by the orientifold, their anti-chiral partners $\bar{\tau}$ correspond to the orthogonal $\mathcal{N} = 1$ subalgebra. The above measure (4.1) does therefore not cover the full $\mathcal{N} = 1$ superspace as required for the generation of a Kähler potential. Rather, the integral is only over half of the $\mathcal{N} = 1$ superspace. While this calls for the generation of an F-term as opposed to a D-term, the additional fermionic zero modes $\bar{\tau}$ will result in higher fermionic couplings of Beasley-Witten type discussed in detail in section 2.3.

An important difference to the F-terms discussed previously is that now only the complex structure moduli receive derivative corrections. Denote by w and a the scalar and axionic parts of the scalar component $U = w - ia$ of a complex structure superfield. Then

¹³Recall that the local factorisation of the moduli space describing the vector and hypermultiplets in general $\mathcal{N} = 2$ compactifications [52] forbids corrections to the Kähler moduli since the dilaton sits in a hypermultiplet.

evaluation of the amplitudes $\langle \theta \bar{w} \bar{\tau} \rangle$ and $\langle \theta \bar{a} \bar{\tau} \rangle$ gives rise to the terms

$$\theta \sigma^\mu \bar{\tau} \partial_\mu \bar{w}^i, \quad \theta \sigma^\mu \bar{\tau} \partial_\mu \bar{a}^i \tag{4.3}$$

in the moduli action. For the details of this computation in the context of toroidal orbifolds see appendix B. The absence of analogous terms for the Kähler moduli is a consequence of U(1) worldsheet charge conservation. Integrating out two copies thereof indeed generates a derivative coupling of the form $e^{-S_{E2}} \partial \bar{U} \partial \bar{U}$. Together with their fermionic partners, the derivative F-terms can be summarized by

$$S = \int d^4x d^2\theta e^{-\mathcal{U}(\Xi)} f_{\bar{i},\bar{j}}(e^{\mathcal{T}_i}, e^{\Delta_i}) \bar{\mathcal{D}}^{\bar{i}} \bar{\mathcal{U}}^{\bar{i}} \bar{\mathcal{D}}_{\bar{\alpha}} \bar{\mathcal{U}}^{\bar{j}} + h.c., \tag{4.4}$$

where the complex conjugate part is due to the anti-instanton contribution. Note the difference to eq. (2.7) describing the higher fermionic terms for $E2$ -instantons with deformation modes. In the presence of charged zero modes λ these F-term corrections for the complex structure moduli involve appropriate powers of charged open string fields required to soak up the λ modes. This amounts to replacing $e^{-\mathcal{U}(\Xi)} \rightarrow e^{-\mathcal{U}(\Xi)} \prod_i \Phi_{a_i, b_i}$.

5. Flux-induced lifting of zero modes

The additional two zero modes $\bar{\tau}^\alpha$ which, if present, prevent the generation of a superpotential by the instanton, are a consequence of the underlying $\mathcal{N} = 2$ supersymmetry preserved in the bulk of the Calabi-Yau away from the orientifold plane in the way described in section 2. It has therefore been speculated in the literature [12, 13, 6] that these Goldstinos might be lifted in the presence of suitable background fluxes. An intuitive reason why this could be the case is that under appropriate circumstances the instanton is expected to feel only the $\mathcal{N} = 1$ supersymmetry preserved by the flux in the bulk. In such situations the $\bar{\tau}$ modes are not protected as the Goldstinos of the orthogonal $\mathcal{N} = 1$ supersymmetry and it might be possible that indeed only the two θ^α modes remain massless in the universal zero modes sector.

While our previous presentation has focused on D-brane instantons in Type IIA orientifolds, the natural arena to study the effects of background fluxes is the framework of Type IIB compactifications, where we can take advantage of the by now quite mature understanding of a fully consistent incorporation of supersymmetric three-form flux (for references see e.g. [53, 54]). The lifting of fermionic zero modes by supersymmetric three-form flux has been analysed in special cases in [28–31] in the context of $E3$ -instantons wrapping a holomorphic divisor of the internal (conformal) Calabi-Yau. The most general such situation involves the presence also of supersymmetric gauge flux on the worldvolume $E3$ -brane. This corresponds on the Type IIA side to $E2$ -instantons at general angles with the $O6$ -plane and is the configuration we are primarily interested in. To the best of our knowledge the possible consequences of such gauge fluxes on the zero mode structure have not been analysed explicitly so far. Before addressing the more general case, we review first the situation of vanishing gauge flux.

5.1 Zero mode lifting for unmagnetised E3-instantons

In the spirit of [55], we consider Type IIB orientifold compactifications with an $\mathcal{N} = 1$ supersymmetric combination $G = F - \tau H$ of RR and NS flux $F = dC_2$ and $H = dB$ such that the complexified dilaton $\tau = C_0 + ie^{-\phi}$ is constant. The internal manifold is therefore conformally Calabi-Yau with constant warp factor. In order to preserve supersymmetry, the flux has to be of (2,1) type¹⁴ and satisfy the primitivity condition $J \wedge G = 0$ in terms of the Kähler form J . We consider an E3-brane wrapping a holomorphic divisor Γ . Since our interest here focuses on the lifting of $\bar{\tau}$ -modes, we assume that Γ is not invariant under the holomorphic involution σ defining the orientifold action $\Omega(-1)^{FL}\sigma$ so that the $\bar{\tau}$ -modes are not projected out. For the simple setup of unmagnetised divisors, we can then simply identify the instanton with its orientifold image and focus on the instanton action before orientifolding without further ado.

The part in the E3-brane worldvolume action describing the coupling of such three-form flux to the (uncharged) zero modes ω ¹⁵ reads [56, 30]

$$S = \int_{\Gamma} d^4\zeta \sqrt{\det g} \omega \left(e^{-\phi} \Gamma^{\tilde{m}} \nabla_{\tilde{m}} + \frac{1}{8} \tilde{G}_{\tilde{m}\tilde{n}p} \Gamma^{\tilde{m}\tilde{n}p} \right) \omega. \tag{5.1}$$

The combination $\tilde{G}_{\tilde{m}\tilde{n}p}$ appearing above is defined as $\tilde{G}_{\tilde{m}\tilde{n}p} = e^{-\phi} H_{\tilde{m}\tilde{n}p} + iF'_{\tilde{m}\tilde{n}p} \gamma_5$ in terms of $F'_{\tilde{m}\tilde{n}p} = F_{\tilde{m}\tilde{n}p} - C_0 H_{\tilde{m}\tilde{n}p}$ and the four-dimensional matrix γ_5 . The indices \tilde{m}, \tilde{n} are along the four-cycle Γ and p is transverse to it. While the above action was derived in [56, 30] entirely with the help of supergravity methods, one could in principle determine it by analysing the CFT coupling of the closed string fields to the boundary, see [57–60] for the relevant techniques.

The Euclidean action (5.1) uses a particular gauge fixing condition to eliminate the unphysical degrees of freedom due to κ -symmetry (cf. eq. 4.9 of [30]). As a result, the spinor ω is a sixteen-component Weyl spinor since we consider a Euclidean action. Locally, we can choose complex coordinates $a, b = 1, 2$ along Γ and z, \bar{z} for the transverse direction. It is convenient to use the standard definition of the Clifford vacuum $|\Omega\rangle$,

$$\Gamma^z |\Omega\rangle = 0, \quad \Gamma^a |\Omega\rangle = 0 \tag{5.2}$$

and to decompose the spinor ω into its external and internal part. The latter can be grouped according to its chirality along the normal bundle of the divisor as

$$\begin{aligned} \epsilon_+ &= \phi |\Omega\rangle + \phi_a \Gamma^{\bar{a}} |\Omega\rangle + \phi_{\bar{a}b} \Gamma^{\bar{a}\bar{b}} |\Omega\rangle, \\ \epsilon_- &= \phi_{\bar{z}} \Gamma^{\bar{z}} |\Omega\rangle + \phi_{\bar{a}\bar{z}} \Gamma^{\bar{a}\bar{z}} |\Omega\rangle + \phi_{\bar{a}\bar{b}\bar{z}} \Gamma^{\bar{a}\bar{b}\bar{z}} |\Omega\rangle. \end{aligned} \tag{5.3}$$

In this language we can immediately identify the universal fermionic zero modes with four-dimensional polarisation θ^α and $\bar{\tau}^\alpha$ as given by

$$\omega_0^{(1)} = \theta \otimes \phi |\Omega\rangle, \quad \omega_0^{(2)} = \bar{\tau} \otimes \phi_{\bar{a}\bar{b}\bar{z}} \Gamma^{\bar{a}\bar{b}\bar{z}} |\Omega\rangle. \tag{5.4}$$

¹⁴In the presence of a non-perturbative superpotential this condition is relaxed to include also (0,3) components [29].

¹⁵The corresponding objects in [30] are called θ , see eq. (4.1.). Recall that we reserve the notation θ and $\bar{\tau}$ for the four-dimensional spinor associated with the universal zero modes.

The fact that they are the "universal" zero modes follows from their correspondence with the cohomology group $H^{(0,0)}(\Gamma)$. The remaining components in (5.3) are associated with the reparametrisation moduli and Wilson line fermions of the four-cycle counted by $H^{(0,2)}(\Gamma)$ and $H^{(0,1)}(\Gamma)$, respectively [61, 30, 62].

Starting from the above action, i.e. in the absence of gauge flux, [30] computed the remaining zero modes in the presence of primitive (2,1) three-form flux. In particular, their analysis shows that the four universal zero modes (5.4) are not lifted in such a situation. In fact, one can easily convince oneself that the zero mode $\omega_0^{(2)}$ does not couple to primitive (2,1) flux. e.g.

$$\begin{aligned} \tilde{G}_{\bar{a}bz} \Gamma^{\bar{a}bz} \Gamma^{\bar{1}} \Gamma^{\bar{2}} \Gamma^{\bar{3}} |\Omega\rangle &= \tilde{G}_{\bar{a}bz} g^{b\bar{1}} g^{z\bar{3}} \Gamma^{\bar{a}} \Gamma^{\bar{2}} |\Omega\rangle - \tilde{G}_{\bar{a}bz} g^{b\bar{2}} g^{z\bar{3}} \Gamma^{\bar{a}} \Gamma^{\bar{1}} |\Omega\rangle \\ &= \tilde{G}_{\bar{1}bz} g^{b\bar{1}} g^{z\bar{3}} \Gamma^{\bar{1}} \Gamma^{\bar{2}} |\Omega\rangle + \tilde{G}_{\bar{2}bz} g^{b\bar{2}} g^{z\bar{3}} \Gamma^{\bar{1}} \Gamma^{\bar{2}} |\Omega\rangle \\ &= \tilde{G}_{\bar{a}bz} g^{b\bar{a}} g^{z\bar{3}} \Gamma^{\bar{1}} \Gamma^{\bar{2}} |\Omega\rangle = 0. \end{aligned} \tag{5.5}$$

The last equation follows from the identity [30]

$$\tilde{G} |\Omega\rangle = i G |\Omega\rangle \tag{5.6}$$

together with primitivity of G ,

$$g^{c\bar{c}'} G_{bc\bar{c}'} = 0. \tag{5.7}$$

Likewise, potential (0,3) components of G-flux can be shown not to couple to the universal modes. This type of flux is allowed by the equations of motion and supersymmetric once the non-perturbative superpotential is taken into account in the analysis of the gravitino variation [63, 29].

5.2 Zero mode lifting for magnetised E3-instantons

We are now ready to address our main question, the inclusion of non-trivial gauge flux on the instanton. The worldvolume action of the $E3$ -instanton contains, in addition to (5.1), two pieces linear and quadratic in the gauge invariant combination $\mathcal{F} = F_{\text{gauge}} - B$ of the worldvolume gauge field and Neveu-Schwarz two-form. Since we are considering an orientifold, we have to add the contribution of the $E3$ -instanton together with its image under $\Omega(-1)^{F_L} \sigma$. As described in [61], this amounts to considering the instanton wrapping the divisor $\tilde{\Gamma} = \Gamma + \sigma\Gamma$ and to expand the worldvolume fields, according to their parity under σ , into their components along the invariant and anti-invariant cohomology on $\tilde{\Gamma}$. Since \mathcal{F} is anti-invariant under Ω , the linear terms in the action survive only for the components of \mathcal{F} along elements of $H_{(1,1)}^-(\tilde{\Gamma})$.

Before orientifolding, the relevant part of the quadratic term is the sum of the two

terms.¹⁶ [28]

$$\begin{aligned}
 S_{\text{DBI}} &= -\frac{\mu}{48} \int_{\Gamma} d^4\zeta \sqrt{\det g} \omega \Gamma^{\tilde{m}\tilde{n}p} \omega e^{-\phi} H_{\tilde{m}\tilde{n}p} \left(\frac{1}{4} \mathcal{F}^2 \right), \\
 S_{\text{WZ}} &= -\frac{\mu}{48} \int_{\Gamma} \omega \Gamma^{\tilde{m}\tilde{n}p} \omega (iF'_{\tilde{m}\tilde{n}p}) \left(\frac{1}{2} \mathcal{F} \wedge \mathcal{F} \right).
 \end{aligned}
 \tag{5.8}$$

In four Euclidean dimensions, solutions to the field equations and Bianchi identity can be taken to satisfy the self-duality constraint $\mathcal{F} = \star\mathcal{F}$. Together with $\int \sqrt{\det g} (\frac{1}{4} \mathcal{F}^2) = \int \frac{1}{2} \mathcal{F} \wedge \star\mathcal{F}$ we find that the relevant couplings combine into

$$-\frac{\mu}{48} \int_{\Gamma} d^4\zeta \sqrt{\det g} \omega G_{\tilde{m}\tilde{n}p} \Gamma^{\tilde{m}\tilde{n}p} \omega \left(\frac{1}{4} \mathcal{F}^2 \right).
 \tag{5.9}$$

By the same reasoning as above, this interaction does not induce any mass terms for the universal zero modes provided we stick to supersymmetric (2,1) (or even (0,3)) flux.

Let us now discuss if the term linear in \mathcal{F} saves the day, given in the upstairs geometry by [28]

$$\begin{aligned}
 S &= \frac{\mu}{16} \int_{\Gamma} d^4\zeta \sqrt{\det g} \left(\mathcal{F}_{\tilde{i}\tilde{k}} \omega \Gamma^{\tilde{k}st} e^{-\phi} H^{\tilde{i}}_{st} \omega - \frac{i}{2} \epsilon^{\tilde{i}jkl} \mathcal{F}_{\tilde{i}j} \omega \Gamma_{\tilde{k}}^{st} (F')_{\tilde{l}st} \omega \right) \\
 &= -i \frac{\mu}{16} \int_{\Gamma} d^4\zeta \sqrt{\det g} \mathcal{F}_{\tilde{i}j} \omega \Gamma^{\tilde{i}st} \omega g^{\tilde{j}k} G_{\tilde{k}st}.
 \end{aligned}
 \tag{5.10}$$

Again self-duality of the gauge flux, $\frac{1}{2} \epsilon^{\tilde{i}jkl} \mathcal{F}_{\tilde{i}j} = \mathcal{F}^{\tilde{k}l}$, is used and a tilde denotes indices parallel to the worldvolume, whereas s, t are general internal indices.

While the index structure of the Γ -matrices is still of type (2,1) due to contraction with the hermitian metric, the above action may in principle induce non-vanishing couplings involving the universal modes. After all, the vanishing of such couplings in the absence of gauge flux rested also upon primitivity of G_3 , which is not necessarily satisfied by the combination of \mathcal{F} and G_3 contracted with the Γ in (5.10). As we stressed, these couplings, being linear in \mathcal{F} , only survive the orientifold action in the presence of anti-invariant two-cycles on the divisor $\tilde{\Gamma}$. We will illustrate this issue in more detail in the next subsection.

On the other hand, in the absence of such cycles, as e.g. for the T^6/\mathbb{Z}_2 example studied in [67], the $\bar{\tau}$ -modes remain massless even after taking into account the backreaction of the three-form flux on the instanton moduli action. While this may seem counter-intuitive because they are no longer protected as Goldstones in the presence of three-form flux, this is just an example of the familiar fact even though all symmetries broken by the instanton result in associated zero modes, the converse need not be true.

¹⁶Note that for simplicity, we are using here the gauge of [28], eq. (29). This κ -symmetry fixing is different from the one in which (5.1) is written and corresponds essentially to the one of [64, 65]. As emphasized in [30, 66] the gauge fixing condition and the orientifold projection have to be compatible for branes invariant under the orientifold. Since we are interested in the more general situation of non-invariant branes or instantons, it suffices for our purposes to work in the gauge of [28].

5.3 A simple example with linear gauge fields

In the presence of suitable three-form flux and for non-vanishing gauge flux F , the linear term in \mathcal{F} leads to a coupling of the zero mode $\omega_0^{(2)}$ proportional to

$$G_{\bar{a}bz} F^{b\bar{a}} g^{z\bar{3}} \Gamma^{\bar{1}} \Gamma^{\bar{2}} |\Omega\rangle. \tag{5.11}$$

As stated above, this does not vanish directly due the primitivity condition for G -flux and the hermitian Yang-Mills equation for the gauge flux F . Under the orientifold projection the flux components $F \in H_{1,1}^+(\tilde{\Gamma})$ are mapped to $-F$ and the associated terms in (5.11) vanish trivially. But for the components $F \in H_{1,1}^-(\tilde{\Gamma})$ there is a chance that the zero modes $\omega_2^{(0)}$ become massive.

More precisely, the action (5.10) leads to a coupling between $\omega_0^{(2)}$ and the mode $\phi_{\bar{a}\bar{b}} \Gamma^{\bar{a}\bar{b}} |\Omega\rangle$. Integrating out the two types of zero modes lifts both the extra universal modes and the deformation modes $\phi_{\bar{a}\bar{b}} \Gamma^{\bar{a}\bar{b}} |\Omega\rangle$. For this mechanism to work, the deformations associated with $\phi_{\bar{a}\bar{b}} \Gamma^{\bar{a}\bar{b}} |\Omega\rangle$ have to be unobstructed, of course. On the other hand, the topological index $N_+ - N_-$ counting the difference between zero modes of positive and negative chirality with respect to the normal bundle of the divisor (see discussion around eq. 5.3) remains unchanged. This is reassuring as by turning on suitable background B-field in addition to the gauge flux we may continuously set the quantity \mathcal{F} appearing in the coupling (5.10) to zero, which should not change any topological quantities.

Let us illustrate this in a simple local example on a toroidal orientifold. We compactify Type IIB on T^6 with metric

$$ds^2 = \sum_I dz_I d\bar{z}_I \tag{5.12}$$

and mod out by the orientifold projection $\Omega\sigma(-1)^{F_L}$ with $\sigma : z_2 \rightarrow -z_2$. Ignoring the resulting tadpole cancellation conditions for a second, we now turn on $\Omega\sigma(-1)^{F_L}$ invariant G_3 -form flux. Consider an $E3$ -brane in this background on the divisor Γ given by the first two T^2 s times a point on the third one. For vanishing Wilson line along the first T^2 , Γ is invariant under the orientifold projection and the instanton is of type $O(1)$. Since we are interested in lifting the $\bar{\tau}$ -modes we assume the presence of a Wilson line rendering the instanton non-invariant.

On this $E3$ -brane we turn on constant gauge flux of type

$$F^{\bar{1}\bar{2}} \in H_{1,1}^-(\tilde{\Gamma}), \tag{5.13}$$

with $\tilde{\Gamma} = \Gamma + \sigma\Gamma$ as before. This flux is invariant under the orientifold projection and satisfies the HYM equation. Consistently, the brane couples to the likewise invariant two-form $(C_2)_{\bar{1},2}$.

Then the coupling of the zero mode $\omega_0^{(2)}$ on the instanton is proportional to

$$G_{\bar{a}bz} F^{\bar{a}\bar{b}} = G_{\bar{1}\bar{2}\bar{3}} F^{\bar{1}\bar{2}}, \tag{5.14}$$

which can be non-vanishing. Indeed the flux component $G_{\overline{1}23}$ is invariant under the orientifold projection. This simple example shows that, ignoring tadpole constraints, it is possible that the $\omega_0^{(2)}$ modes decouple for non-vanishing G_3 form flux.

However, when it comes to satisfying the tadpole constraints, we have to introduce both further $D7$ -branes to cancel the $O7$ -plane tadpole and an $O3$ -plane to cancel the tadpole induced by the G_3 -form. The easiest way to get the $O3$ -plane is to also mod out the model by the \mathbb{Z}_2 action $z_{1,3} \rightarrow -z_{1,3}$, essentially turning the configuration into the fluxed $K3 \times \frac{T^2}{\mathbb{Z}_2}$ model studied in [68]. However, in this case the $E3$ is not invariant under this \mathbb{Z}_2 , but mapped to an $E3$ brane with opposite gauge flux $-F^{\overline{1}2}$. Therefore, the coupling of the $\omega_0^{(2)}$ modes again trivially vanishes. We leave it for future work to study more general concrete global models of such a configuration in detail and to verify if the $\overline{\tau}$ -modes can actually be lifted.

6. Conclusions

This paper has investigated in detail under which circumstances D-brane instantons can contribute to the superpotential in Type II orientifolds. A key role is played by the two universal zero modes $\overline{\tau}$ which are a remnant of the local $\mathcal{N} = 2$ supersymmetry felt by instantons not invariant under the orientifold action. Their presence obstructs the generation of a superpotential. If these modes are not lifted and in the absence of additional zero modes between the instanton and its orientifold image, the instanton generates higher-fermionic F-term corrections which in general depend also on open string operators. Previously, such terms had been considered in the context of heterotic worldsheet instantons moving in a family [25].

Our main interest has been in possible mechanisms to lift the $\overline{\tau}$ modes such that superpotential contributions are possible. Clearly, this question is of significance for an analysis of the quantum corrected moduli space of string vacua as well as for determining the effective interactions in the vacuum.

We first focused on an effect which, for $E2$ -instantons in Type IIA orientifolds, is describable as recombination of the instanton with its orientifold image. Equivalently, we asked whether in the Type IIB/Type I dual picture $E5$ -instantons carrying non-trivial extension bundles generate superpotential couplings. If so, this would have important consequences also for the heterotic string. We found that while the $\overline{\tau}$ -modes are indeed absent in such situations, there arise generically additional charged zero modes which cannot be lifted, thus obstructing a contribution to the effective action. By contrast, for the special case that the instanton and its orientifold image preserve a common $\mathcal{N} = 1$ supersymmetry, no such zero modes arise and the recombined object can generate a superpotential provided its reparametrisation moduli can be lifted. For general Calabi-Yau manifolds, we identified appropriate open-string dependent couplings in the instanton moduli action. Their presence hinges upon the details of the underlying $\mathcal{N} = (2, 2)$ superconformal worldsheet theory. These couplings generalise known examples of the lifting of instanton reparameterisation moduli through curvature couplings or background fluxes.

Concerning this latter point, we tried to substantiate the well-motivated speculation [12, 13, 6] that closed string background fluxes might also lift the universal $\bar{\tau}$ modes, restricting ourselves to the familiar framework of Type IIB orientifolds with supersymmetric three-form flux. In agreement with the results in particular of [30], in the absence of gauge flux on the $E3$ -instanton no such lifting occurs. We showed, building on the instanton action derived in [28], that once worldvolume fluxes are turned, a lifting might be possible, but only in situations where the divisor wrapped by the instanton contains non-trivial two-cycles anti-invariant under the orientifold action. As it stands we have to leave it open whether this effect can actually be realised in explicit models and, if so, whether it enables the instanton to contribute to the superpotential. As one of the most imminent open questions it therefore remains to study a concrete global example in the spirit of the setup discussed in the last section. Also, it would be desirable to gain comparable understanding of the effects of Type IIA fluxes on the $E2$ -instanton zero modes.

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A. Orientifold projection of instanton zero modes

In this appendix we describe explicitly the orientifold action $\Omega\bar{\sigma}$ on the zero modes of an $E2$ -instanton wrapping the cycle Ξ . If Ξ is not invariant under the orientifold action one includes, in the upstairs picture, the orientifold image $E2'$ wrapping the image cycle Ξ' . The orientifold action identifies the $E2-E2$ modes with the $E2'-E2'$ modes and $E2-E2'$ -modes with $E2'-E2$ -modes. The $E2-E2'$ modes arising at invariant intersections on top of the orientifold plane are symmetrised/anti-symmetrised as will be described momentarily. The same applies to the $E2-E2$ sector if the $E2$ wraps a cycle invariant under the orientifold action, $\Xi = \Xi'$.

The orientifold action on the bosonic and fermionic instanton zero modes in the invariant sector can be deduced from the action on spacetime-filling $D6$ -branes wrapping the same internal cycle Ξ (and possibly its image) as follows:

1. The orientifold action on the internal oscillator part of the vertex operators agrees in the $D6$ and $E2$ case. The only difference in the $E2$ case is that the external 4D space is orthogonal to the $E2$ -brane and thus counts as transverse when applying the usual rules for representing $\Omega\bar{\sigma}$. This entails the inclusion of an additional minus sign for bosonic excitations in the external 4D space and the inclusion of a factor $e^{i\pi(s_0+s_1)}$

for all fermionic zero modes. Here $e^{i\pi(s_0+s_1)}$ acts on the (anti-)chiral 4D spin fields S^α ($S^{\dot{\alpha}}$) as $e^{i\pi(s_0+s_1)}S^\alpha = -1$ ($e^{i\pi(s_0+s_1)}S^{\dot{\alpha}} = 1$).

- Let $\gamma_{\Omega\bar{\sigma},D6}$ denote the matrix representing the orientifold action on the CP factors of the D6-brane modes. Then the corresponding matrix for the E2-instanton $\gamma_{\Omega\bar{\sigma},E2}$ enjoys

$$\gamma_{\Omega\bar{\sigma},D6} = \pm \gamma_{\Omega\bar{\sigma},D6}^T \iff \gamma_{\Omega\bar{\sigma},E2} = \mp \gamma_{\Omega\bar{\sigma},E2}^T. \tag{A.1}$$

The + and - cases for the projection relevant for D6-branes are referred to as orthogonal (SO) and symplectic (SP) projections, respectively, because for invariant D6-branes they yield gauge bosons in the adjoint of the respective gauge groups. In the latter case, invariant cycles have to be wrapped by an even number of D6-branes.

Finally, the relation (A.1) follows via T-duality from the D9-D5 system analysed by Gimon-Polchinski [69].

It is straightforward to apply these rules to the zero modes for two different cases: (i) the universal zero modes for $\Pi_\Xi = \Pi_{\Xi'}$ and (ii) the modes in the $E2 - E2'$ sector arising on top of the orientifold for $\Pi_\Xi \neq \Pi_{\Xi'}$. In case the instanton wraps a cycle $\Pi_\Xi = \Pi_{\Xi'}$ the orientifold action on the universal zero modes x^μ and $\theta^\alpha, \bar{\tau}^{\dot{\alpha}}$ leads to

$$\Omega_{x^\mu} = \gamma_{E2} \Omega_{x^\mu}^T \gamma_{E2}^{-1}, \tag{A.2}$$

$$\Omega_{\theta^\alpha} = \gamma_{E2} \Omega_{\theta^\alpha}^T \gamma_{E2}^{-1}, \quad \Omega_{\bar{\tau}^{\dot{\alpha}}} = -\gamma_{E2} \Omega_{\bar{\tau}^{\dot{\alpha}}}^T \gamma_{E2}^{-1}, \tag{A.3}$$

where for x^μ and θ^α the minus sign due to the excitation gets cancelled by the minus sign due to rule (1). Thus for a single instanton subject to the projection $\gamma_{E2} = \gamma_{E2}^T$, only x^μ and θ^α survive. Modes in the $E2 - E2'$ -sector arising at intersection on top of the orientifold get (anti-)symmetrised as follows:

- If for D6-branes wrapping the same cycle the invariant states get anti-symmetrised, then

$$\Omega_{m/\bar{m}} = \Omega_{m/\bar{m}}^T, \quad \Omega_{\bar{\mu}} = \Omega_{\bar{\mu}}^T, \tag{A.4}$$

$$\Omega_{\mu} = -\Omega_{\mu}^T. \tag{A.5}$$

This results in the intersection numbers displayed in table 4. In particular, for a single instanton, the zero mode μ^α gets projected out and only $m, \bar{m}, \bar{\mu}^{\dot{\alpha}}$ survive corresponding to *case I* in section 3.1

- If for D6-branes the invariant states are symmetrised, everything just changes sign.

B. Details of the CFT computations

In this appendix we demonstrate the computation of the amplitude $\langle m \bar{\tau} \bar{\mu} \rangle$ as well as of some of the couplings of the fermionic zero modes of the instanton to the closed string

background relevant for the F-term corrections investigated in section 2.3 and 4. For simplicity we focus on the case of an instanton wrapping a factorizable cycle of a toroidal orbifold. More details of the CFT computation in this context can be found in [15]. While for other backgrounds the presence of the couplings in question has to be checked in concrete computations, all couplings which do not violate any of the general selection rules of the $\mathcal{N} = 2$ SCFT on the worldsheet are generically present.

Let us start with the open string coupling $\langle m \bar{\tau} \bar{\mu} \rangle$ used in eq. 3.12. The relevant vertex operators take the form¹⁷

$$\begin{aligned}
 V_{\bar{\tau}}^{(-\frac{1}{2})}(z) &= \Omega_{\bar{\tau}} \bar{\tau}^{\dot{\alpha}} S_{\dot{\alpha}}(z) \prod_{i=1}^3 e^{-i/2 H_i(z)} e^{-\varphi/2(z)} \\
 V_m^{(-1)}(z) &= \Omega_m m \prod_{i=1}^3 e^{i(1-\theta_{E2E2'}^i) H_i(z)} e^{-\varphi(z)} \\
 V_{\bar{\mu}}^{(-\frac{1}{2})}(z) &= \Omega_{\bar{\mu}} \bar{\mu}^{\dot{\alpha}} S_{\dot{\alpha}}(z) \prod_{i=1}^3 e^{-i(\frac{1}{2}-\theta_{E2E2'}^i) H_i(z)} e^{-\varphi(z)}.
 \end{aligned} \tag{B.1}$$

Inserting them into $\langle m \bar{\tau} \bar{\mu} \rangle$ leads to

$$\begin{aligned}
 \langle m \bar{\tau} \bar{\mu} \rangle &= Tr(\Omega_m \Omega_{\bar{\tau}} \Omega_{\bar{\mu}}) m \bar{\tau}^{\dot{\alpha}} \bar{\mu}^{\dot{\beta}} \langle e^{-\varphi(z_1)} e^{\varphi/2(z_2)} e^{-\varphi/2(z_3)} \rangle \\
 &< S_{\dot{\alpha}}(z_1) S_{\dot{\beta}}(z_3) \rangle \prod_{i=1}^3 \langle e^{i(1-\theta_{E2E2'}^i) H_i(z_1)} e^{-i/2 H_i(z_2)} e^{-i(\frac{1}{2}-\theta_{E2E2'}^i) H_i(z_3)} \rangle,
 \end{aligned}$$

which together with the supersymmetry condition $\sum_{i=1}^3 \theta_{E2E2'}^i = 2$ results in the coupling

$$m \bar{\tau}_{\dot{\alpha}} \bar{\mu}^{\dot{\alpha}}. \tag{B.2}$$

Now we turn to interactions between fermionic zero modes of the instanton and closed string background fields. We start with the coupling between the reparametrization moduli $\bar{\chi}^{\dot{\alpha}}$ surviving the orientifold action and the anti-chiral Kähler moduli $\bar{t}^{\dot{\alpha}}$. Their vertex operator in type IIA takes the form

$$\begin{aligned}
 V_{\bar{\chi}^I}^{(-\frac{1}{2})}(z) &= \Omega_{\bar{\chi}} \bar{\chi}^{I\dot{\alpha}} e^{-\varphi/2(z)} S_{\dot{\alpha}}(z) e^{-i/2 H_I(z)} \prod_{i \neq I}^3 e^{i/2 H_i(z)}, \\
 V_{\bar{t}^{IJ}}^{(-\frac{1}{2}, -1)}(z) &= \bar{t}^{IJ\dot{\alpha}} e^{-\varphi/2(z)} S_{\dot{\alpha}}(z) e^{-i/2 H_I(z)} \prod_{i \neq I}^3 e^{i/2 H_i(z)} e^{-\tilde{\varphi}(\bar{z})} e^{-i\tilde{H}_J(\bar{z})(\bar{z})} e^{ikX(z, \bar{z})}.
 \end{aligned} \tag{B.3}$$

$$\tag{B.4}$$

Note that on a factorizable torus $T^6 = T^2 \times T^2 \times T^2$ only the diagonal moduli \bar{t}^{JJ} survive.

We see that the couplings $\langle \bar{\chi} \bar{t} \rangle$ respect the total U(1) worldsheet charge. However, only the amplitudes $\langle \bar{\chi}^I \bar{t}^{JK} \rangle$ for $I \neq J \neq K \neq I$ preserve the internal U(1)-charge in each T^2 separately and lead to a coupling

$$\bar{\chi}_{\dot{\alpha}}^I \bar{t}^{JK\dot{\alpha}}. \tag{B.5}$$

¹⁷Here we assume the most symmetric configuration in which all intersection angles $\theta_{E2E2'}^i > 0$ and $\sum_{i=1}^3 \theta_{E2E2'}^i = 2$.

While on factorizable tori $(T^2)^3$ and orbifolds thereof no such couplings exist, on general Calabi-Yau threefolds there is no reason for them to vanish.

On the other hand, one can convince oneself that the anti-chiral complex structure moduli with vertex operators

$$V_{\bar{u}^{IJ}}^{(-\frac{1}{2}, -1)} = \bar{u}^{IJ\dot{\alpha}} e^{-\varphi/2(z)} S_{\dot{\alpha}}(z) e^{-i/2 H_I(z)} \prod_{i \neq I}^3 e^{i/2 H_i(z)} e^{-\tilde{\varphi}(\bar{z})} e^{i\tilde{H}_J(\bar{z})}(\bar{z}) e^{ikX(z, \bar{z})} \quad (\text{B.6})$$

do not couple to $\bar{\chi}$ due to non conservation of the total U(1) world sheet charge. This is therefore a universal result.

The corresponding bosonic superpartner terms to (B.5) arise from amplitudes of the form

$$\langle \theta^{(+1/2)} \bar{T}^{(-1, -1)} \bar{\chi}^{(-1/2)} \rangle, \quad (\text{B.7})$$

where the superscripts denote ghost picture of the respective vertex operator. Note that with the choice displayed in (B.7) we ensure the total ghost charge constraint. The vertex operator of the Kähler moduli takes the form

$$V_{\bar{T}^{IJ}}^{(-1, -1)} = \bar{T}^{IJ} e^{-\varphi(z)} e^{-iH_I(z)} e^{-\tilde{\varphi}(\bar{z})} e^{-i\tilde{H}_J(\bar{z})} e^{ikX(z, \bar{z})}$$

while the one for the θ -mode in $(+\frac{1}{2})$ -ghost picture is given by

$$V_{\theta^\alpha}^{(+\frac{1}{2})}(z) = \Omega_\theta \theta^\alpha \left[\partial X_\mu (\sigma^\mu)_\alpha^{\dot{\alpha}} S_{\dot{\alpha}}(z) \prod_{i=1}^3 e^{i/2 H_i(z)} + \sum_{I=1}^3 S_\alpha(z) \partial Z^I e^{-i/2 H_I(z)} \prod_{i \neq I}^3 e^{i/2 H_i(z)} \right] e^{\varphi/2(z)}. \quad (\text{B.8})$$

By internal U(1) charge conservation only the first summand contributes to the amplitude and in addition one has to require, as for $\langle \bar{\chi} \bar{t} \rangle$, that $I \neq J \neq K \neq I$. Then

$$\begin{aligned} \langle \theta \bar{T}^{JK} \bar{\chi}^I \rangle &= Tr(\Omega_\theta \Omega_{\bar{\chi}}) \theta^\alpha (\sigma^\mu)_\alpha^{\dot{\alpha}} \bar{\chi}^I \bar{T}^{JK} \\ &\langle e^{\varphi/2(z_1)} e^{-\varphi(z_2)} e^{-\tilde{\varphi}(\bar{z}_2)} e^{-\varphi/2(z_3)} \rangle \langle S_{\dot{\alpha}}(z_1) S_{\dot{\beta}}(z_3) \rangle \\ &\langle e^{i/2 H_I(z_1)} e^{-i/2 H_I(z_3)} \rangle \langle e^{i/2 H_J(z_1)} e^{-i H_J(z_2)} e^{i/2 H_J(z_3)} \rangle \\ &\langle e^{i/2 H_K(z_1)} e^{-i\tilde{H}_K(\bar{z}_2)} e^{i/2 H_K(z_3)} \rangle \langle \partial X_\mu(z_1) e^{ik_2 X(z_2, \bar{z}_2)} \rangle. \end{aligned}$$

The correlators are easily evaluated and lead to couplings proportional to

$$\theta \sigma^\mu \bar{\chi}^I \partial_\mu \bar{T}^{JK}.$$

As above, by non-conservation of U(1)-charge there is no coupling to the bosonic complex structure field U . On the other hand the amplitude $\langle \theta u^{IJ} \rangle$ is non-vanishing. Here the vertex operator of u^{IJ} is the complex conjugated of (B.6)

$$V_{u^{IJ}}^{(-\frac{1}{2}, -1)}(z) = u_\alpha^{IJ} e^{-\varphi/2(z)} S^\alpha(z) e^{i/2 H_I(z)} \prod_{i \neq I}^3 e^{-i/2 H_i(z)} e^{-\tilde{\varphi}(\bar{z})} e^{-i\tilde{H}_J(\bar{z})}(\bar{z}) e^{ikX(z, \bar{z})}$$

and the vertex operator for θ in $(-\frac{1}{2})$ -ghost picture takes the form

$$V_{\theta}^{(-\frac{1}{2})}(z) = \Omega_{\theta} \theta_{\alpha} S^{\alpha}(z) \prod_{i=1}^3 e^{i/2 H_i(z)} e^{-\varphi/2(z)}. \quad (\text{B.9})$$

Now, one can easily check that U(1) world sheet charge is conserved only in case $I = J$ and the resulting coupling takes the form

$$\theta^{\alpha} u_{\alpha}^{\text{II}}. \quad (\text{B.10})$$

For the couplings relevant in section 4 we also need the corresponding bosonic partner arise from amplitudes involving a^{IJ} and $\bar{\omega}^{\text{IJ}}$. For brevity we only display the computation of the amplitude $\langle \theta^{(1/2)} \bar{\omega}^{(-1,-1)} \bar{\tau}^{(-1/2)} \rangle$, where the vertex operator for $\bar{\omega}$ is

$$V_{\bar{\omega}^{\text{IJ}}}^{(-1,-1)}(z) = \bar{\omega}^{\text{IJ}} e^{-\varphi(z)} e^{iH_I(z)} e^{-\bar{\varphi}(\bar{z})} e^{-i\bar{H}_J(\bar{z})} e^{ikX(z,\bar{z})}, \quad (\text{B.11})$$

while the vertex operator for the θ and $\bar{\tau}$ in the respective ghost picture are given by (B.8) and (B.1). Again U(1) world sheet charge requires $I = J$ and a computation analogous to the one leading to the amplitude $\langle \theta \bar{T} \bar{\chi} \rangle$ gives the coupling

$$\theta \sigma^{\mu} \bar{\tau} \partial_{\mu} \bar{\omega}^{\text{II}}. \quad (\text{B.12})$$

On the other hand due to the U(1) world-sheet charge there the amplitudes $\langle \bar{\tau} \bar{t} \rangle$ as well as $\langle \theta \bar{T} \bar{\tau} \rangle$ vanish.

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